Principal filters of some ordered \( \Gamma \)-semigroups

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Abstract. For an intra-regular or a left regular and left duo ordered \( \Gamma \)-semigroup \( M \), we describe the principal filter of \( M \) which plays an essential role in the structure of this type of \( po- \) \( \Gamma \)-semigroups. We also prove that an ordered \( \Gamma \)-semigroup \( M \) is intra-regular if and only if the ideals of \( M \) are semiprime and it is left (right) regular and left (right) duo if and only if the left (right) ideals of \( M \) are semiprime.

Key Words: ordered \( \Gamma \)-semigroup, filter, intra-regular, left regular

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1 Introduction and prerequisites

Our aim is to describe the principal filters of intra-regular ordered \( \Gamma \)-semigroups and the principal filters of ordered \( \Gamma \)-semigroups which are both left regular and left duo. Croisot, who used the term “inversive” instead of “regular”, connects the matter of decomposition of a semigroup with the regularity and semiprime conditions [2]. A semigroup \( S \) is said to be left (resp. right) regular if for every \( a \in S \) there exists \( x \in S \) such that \( a = xa^2 \) (resp. \( a = a^2x \)). That is, if \( a \in Sa^2 \) (resp. \( a \in a^2S \)) for every \( a \in S \) which is equivalent to saying that \( A \subseteq A^2S \) (resp. \( A \subseteq SA^2 \)) for every \( A \subseteq S \). A semigroup \( S \) is said to be intra-regular if for every \( a \in S \) there exist \( x, y \in S \) such that \( a = xa^2y \). In other words, if \( a \in Sa^2S \) for every \( a \in S \) or \( A \subseteq SA^2S \) for every \( A \subseteq S \). For decompositions of an intra-regular, of a left regular or both left and right regular semigroup we refer to [1, 7]. The concepts of intra-regular ordered semigroup and of right regular ordered semigroup have been introduced in [3, 4] in which the decomposition of an intra-regular ordered semigroup into simple components and the decomposition of a right regular and right duo ordered semigroup into right simple components have been studied. The principal filter of \( S \) has a very simple form for both ordered and unordered case of \( \Gamma \)-semigroups, and it plays an essential role in their decomposition.
For the sake of completeness, let us first give the definition of a \( \Gamma \)-semigroup. In this paper we use the definition of \( \Gamma \)-semigroup introduced by Saha in [8]: Given two nonempty sets \( M \) and \( \Gamma \), \( M \) is called a \( \Gamma \)-semigroup if there exists a mapping \( M \times \Gamma \times M \rightarrow M \) \((a, \gamma, b) \rightarrow a\gamma b\) such that \( (a\gamma b)\mu c = a\gamma (b\mu c) \) for every \( a, b, c \in M \) and every \( \gamma, \mu \in \Gamma \). An ordered \( \Gamma \)-semigroup (shortly, po-\( \Gamma \)-semigroup) is clearly a \( \Gamma \)-semigroup \( M \) with an order relation “\( \leq \)” on \( M \) such that \( a \leq b \) implies \( a\gamma c \leq b\gamma c \) and \( c\gamma a \leq c\gamma b \) for every \( c \in M \) and every \( \gamma \in \Gamma \). For a subset \( H \) of \( M \) we denote by \( (H) \) the subset of \( M \) defined by

\[
(H) = \{t \in M \mid t \leq a \text{ for some } t \in H\}.
\]

We mention the properties we use in the paper: Clearly \( M = (M) \), and for any subsets \( A, B, C \) of \( M \), we have the following: \( A \subseteq (A) \); if \( A \subseteq B \), then \( A\Gamma C \subseteq B\Gamma C \) and \( C\Gamma A \subseteq C\Gamma B \); if \( A \subseteq B \), then \( (A) \subseteq (B) \); \( (A\Gamma B) \subseteq (A\Gamma B) \); \( (A\Gamma B) = (A\Gamma B) \); if \( a \leq b \), then \( A\Gamma a \subseteq (A\Gamma b) \) and \( a\Gamma A \subseteq (b\Gamma A) \); \( (A) = (A) \). Let us prove the last one: Since \( A \subseteq (A) \), we have \( (A) \subseteq (A) \). Let now \( t \in (A) \). Then \( t \leq x \) for some \( x \in (A) \) and \( x \leq a \) for some \( a \in A \). Since \( t \in S \) and \( t \leq a \), where \( a \in A \), we have \( t \in (A) \). As one can easily see, the following are equivalent: (1) \( a \in A \) and \( S \ni b \leq a \), then \( b \in A \). (2) \( A \subseteq A \). (3) \( A = A \). A nonempty subset \( A \) of \( M \) is called a subsemigroup of \( M \) if, for every \( a, b \in A \) and every \( \gamma \in \Gamma \), we have \( a\gamma b \in A \), that is if \( A\Gamma A \subseteq A \). A nonempty subset \( A \) of \( M \) is called a left (resp. right) ideal of \( M \) if (1) \( M\Gamma A \subseteq A \) (resp. \( A\Gamma M \subseteq A \)) and (2) if \( a \in A \) and \( M \ni b \leq a \), then \( b \in A \) (equivalently \( A = A \)). It is called an ideal (or two-sided ideal) of \( M \) if it is both a left and right ideal of \( M \). Clearly every left (resp. right) ideal of \( M \) is a subsemigroup of \( M \). A po-\( \Gamma \)-semigroup \( M \) is called left (resp. right) duo if the left (resp. right) ideals of \( M \) are two-sided. A subsemigroup \( F \) of \( M \) is called a filter of \( M \) if (1) for every \( a, b \in M \) and every \( \gamma \in \Gamma \) such that \( a\gamma b \in F \), we have \( a \in F \) and \( b \in F \) and (2) if \( a \in F \) and \( M \ni b \geq a \), then \( b \in F \). For an element \( x \) of \( M \), we denote by \( N(x) \) the filter of \( M \) generated by \( x \) (that is, the least with respect to the inclusion relation filter of \( M \) containing \( x \)). A subset \( T \) of \( M \) is called semiprime if \( x \in M \) and \( \gamma \in \Gamma \) such that \( x\gamma x \in T \) implies \( x \in T \).

As we know, many results on semigroups (ordered semigroups) can be transferred into \( \Gamma \)-semigroups (po-\( \Gamma \)-semigroups) just putting a Gamma in the appropriate place, while for some other results the transfer needs subsequent technical changes. A \( \Gamma \)-semigroup \( M \) is called intra-regular if \( a \in M\Gamma a\Gamma M \) for every \( a \in M \), equivalently if \( A \subseteq M\Gamma A\Gamma M \) for every \( A \subseteq M \). It is called left (resp. right) regular if \( a \in M\Gamma a \Gamma a \Gamma M \) (resp. \( a \in a\Gamma a \Gamma M \)) for every \( a \in M \), equivalently if \( A \subseteq M\Gamma A \Gamma A \) (resp. \( A \subseteq A\Gamma A \Gamma M \)) for every \( a \in M \).
every $A \subseteq M$. An ordered $\Gamma$-semigroup $M$ is called \textit{intra-regular} if for every $a \in M$ we have $a \in (MG\alpha\Gamma M]$, equivalently if for every $A \subseteq M$ we have $A \subseteq (MG\Gamma A\Gamma M]$. It is called \textit{left} (resp. \textit{right}) \textit{regular} if $a \in (MG\alpha\Gamma a]$ (resp. $(a \in (a\Gamma\alpha M]$) for every $a \in M$, equivalently if $A \subseteq (MG\Gamma A\Gamma M] \subseteq (a\Gamma\alpha M]$) for every $A \subseteq M$. Although some interesting results on $\Gamma$-semigroups are obtained using the definition of \textit{left} (resp. \textit{right}) regular or the definition of \textit{intra-regular} ordered $\Gamma$-semigroup mentioned above, with these definitions one fails to prove basic results of $\Gamma$-semigroups, such as to describe the filter of $M$ generated by an element $a$ of $M$, for example, which plays an essential role in the investigation. To overcome this difficulty, a new definition of \textit{intra-regular} and a new definition of left regular $\Gamma$-semigroups has been introduced in [5]. The \textit{intra-regular} $\Gamma$-semigroup has been defined as a $\Gamma$-semigroup $M$ such that $a \in MG\alpha\gamma a\Gamma M$ for each $a \in M$ and each $\gamma \in \Gamma$ and the \textit{left} (resp. \textit{right}) \textit{regular} $\Gamma$-semigroup as a $\Gamma$-semigroup in which $a \in MG\alpha\gamma a$ (resp. $a \in a\gamma a\Gamma M$) for each $a \in M$ and each $\gamma \in \Gamma$ and it is proved that a $\Gamma$-semigroup $M$ is left regular (in that new sense) if and only if it is a union of a family of left simple subsemigroups on $M$. And in [6] we gave some further structure theorems of this type of $\Gamma$-semigroups using that new definition and the form of their principal filters. But what happens in case of \textit{intra-regular} or in case of \textit{left regular} or for \textit{right regular} po-$\Gamma$-semigroups? Can we describe the form of their principal filters using some new definitions similar to the unordered case? The present paper gives the related answer.

2 On intra-regular ordered po-$\Gamma$-semigroups

We characterize here the intra-regular po-$\Gamma$-semigroups in terms of filters, and we prove that a po-$\Gamma$-semigroup $M$ is \textit{intra-regular} if and only if the ideals of $M$ are semiprime.

\textbf{Definition 1.} An ordered $\Gamma$-semigroup $M$ is called \textit{intra-regular} if

$$x \in (MGx\gamma x\Gamma M]$$

for every $x \in M$ and every $\gamma \in \Gamma$.

\textbf{Definition 2.} (cf. also [3]) If $M$ is an ordered $\Gamma$-semigroup, a subset $A$ of $M$ is called \textit{semiprime} if

$$a \in M \text{ and } \gamma \in \Gamma \text{ such that } a\gamma a \in A \text{ implies } a \in A.$$ 

\textbf{Theorem 3.} An ordered $\Gamma$-semigroup $M$ is \textit{intra-regular} if and only if, for every $x \in M$, we have

$$N(x) = \{y \in M \mid x \in (MGy\Gamma M]\}.$$
Proof. \( \Rightarrow \). Let \( x \in M \) and \( T := \{ y \in M \mid x \in (M \Gamma y \Gamma M) \} \). Then we have the following:

(1) \( T \) is a nonempty subset of \( M \). Indeed: Take an element \( \gamma \in \Gamma \) \((\Gamma \neq \emptyset)\). Since \( M \) is intra-regular, we have

\[
x \in (M \Gamma x \gamma x \Gamma M) = ( (M \Gamma x) \gamma x \Gamma M) \subseteq ( (M \Gamma M) \Gamma x \Gamma M) \subseteq (M \Gamma x \Gamma M),
\]

so \( x \in T \).

(2) Let \( a, b \in T \) and \( \gamma \in \Gamma \). Then \( a \gamma b \in T \). Indeed: Since \( a \in T \), we have \( x \in (M \Gamma a \Gamma M) \). Since \( b \in T \), we have \( x \in (M \Gamma b \Gamma M) \). Since \( M \) is intra-regular, \( x \in M \) and \( \gamma \in \Gamma \), we have \( x \in (M \Gamma x \gamma x \Gamma M) \). Then we have

\[
x \in (M \Gamma x \gamma x \Gamma M) \subseteq (M \Gamma (M \Gamma b \Gamma M) \gamma (M \Gamma a \Gamma M) \Gamma M)
= (M \Gamma (M \Gamma b \Gamma M) \gamma (M \Gamma a \Gamma M) \Gamma M)
= (M \Gamma (M \Gamma M \gamma (M \Gamma a) \Gamma (M \Gamma M))
\subseteq (M \Gamma (b \Gamma M \gamma (M \Gamma a) \Gamma M)),
\]

so \( a \gamma b \in T \). Let now \( b \lambda u \gamma v \delta a \in b \Gamma \Gamma M \Gamma a \) for some \( u, v \in M, \lambda, \delta \in \Gamma \). Since \( M \) is intra-regular, for the elements \( b \lambda u \gamma v \delta a \in M \) and \( \gamma \in \Gamma \), we have

\[
b \lambda u \gamma v \delta a \in (M \Gamma (b \lambda u \gamma v \delta a) \gamma (b \lambda u \gamma v \delta a) \Gamma M)
= (M \Gamma (b \lambda u \gamma v) \delta (a \gamma b) \lambda (u \gamma v \delta a \Gamma M))
\subseteq (M \Gamma (a \gamma b) \Gamma M).
\]

(3) Let \( a, b \in M \) and \( \gamma \in \Gamma \) such that \( a \gamma b \in T \). Then \( a, b \in T \). Indeed: Since \( a \gamma b \in T \), we have \( x \in (M \Gamma (a \gamma b) \Gamma M) \subseteq (M \Gamma a \gamma (M \Gamma M)) \subseteq (M \Gamma a \Gamma M) \), so \( a \in T \). Since \( x \in (M \Gamma (a \gamma b) \Gamma M) \subseteq ((M \Gamma M) \gamma b \Gamma M) \subseteq (M \Gamma b \Gamma M) \), we have \( b \in T \).

(4) Let \( a \in T \) and \( M \ni b \geq a \). Then \( b \in T \). Indeed: Since \( a \in T \), we have \( x \in (M \Gamma a \Gamma M) \). Since \( a \leq b \), we have \( M \Gamma a \Gamma M \subseteq (M \Gamma b \Gamma M) \), then \( (M \Gamma a \Gamma M) \subseteq (M \Gamma b \Gamma M) = (M \Gamma b \Gamma M) \). Then we have \( x \in (M \Gamma b \Gamma M) \), and \( b \in T \).
(5) Let $F$ be a filter of $M$ such that $x \in F$. Then $T \subseteq F$. Indeed: Let $a \in T$. Then $x \in (M\Gamma a \Gamma M)$, so $F \ni x \leq u\lambda(a\mu v)$ for some $u, v \in M$, $\lambda, \mu \in \Gamma$. Since $F$ is a filter of $M$, $x \in F$ and $M \ni u\lambda(a\mu v) \geq x$, we have $u\lambda(a\mu v) \in F$. Since $F$ is a filter of $M$, $u, a\mu v \in M$, $\lambda \in \Gamma$ and $u\lambda(a\mu v) \in F$, we have $a\mu v \in F$, again since $F$ is a filter of $M$, $a, v \in M$ and $\mu \in \Gamma$, we have $a \in F$.

$\iff$. Let $x \in M$ and $\gamma \in \Gamma$. Then $x \in (M\Gamma x\gamma x\Gamma M)$. Indeed: Since $N(x)$ is a subsemigroup of $M$, $x \in N(x)$ and $\gamma \in \Gamma$, we have $x\gamma x \in N(x)$. Then, by hypothesis, we get $x \in (M\Gamma(x\gamma x)\Gamma M) = (M\Gamma x\gamma x\Gamma M)$, thus $M$ is intra-regular. $\square$

**Theorem 4.** An ordered $\Gamma$-semigroup $M$ is intra-regular if and only if the ideals of $M$ are semiprime.

**Proof.** $\implies$. Let $A$ be an ideal of $M$, $x \in M$ and $\gamma \in \Gamma$ such that $x\gamma x \in A$. Since $M$ is intra-regular, we have

$$x \in (M\Gamma(x\gamma x)\Gamma M) \subseteq (M\Gamma A)\Gamma M \subseteq (A\Gamma M) \subseteq (A) = A,$$

then $x \in A$, and $A$ is semiprime.

$\iff$. Let $x \in M$ and $\gamma \in \Gamma$. Then $x \in (M\Gamma x\gamma x\Gamma M)$. In fact: The set $(M\Gamma x\gamma x\Gamma M)$ is an ideal of $M$. This is because it is a nonempty subset of $M$, $M\Gamma(M\Gamma x\gamma x\Gamma M) \subseteq (M\Gamma(M\Gamma x\gamma x\Gamma M)) = (M\Gamma(M\Gamma x\gamma x\Gamma M)) \subseteq (M\Gamma x\gamma x\Gamma M)$, $(M\Gamma x\gamma x\Gamma M)\Gamma M \subseteq (M\Gamma x\gamma x\Gamma M)$, and $(M\Gamma x\gamma x\Gamma M) = (M\Gamma x\gamma x\Gamma M)$ (since this holds for any subset $A$ of $M$). Since $(M\Gamma x\gamma x\Gamma M)$ is semiprime, $x\gamma x \in M\Gamma M \subseteq M$, $\gamma \in \Gamma$ and

$$(x\gamma x)\gamma(x\gamma x) = x\gamma(x\gamma x)\gamma x \in M\Gamma x\gamma x\Gamma M \subseteq (M\Gamma x\gamma x\Gamma M),$$

we have $x\gamma x \in (M\Gamma x\gamma x\Gamma M)$. Then, since $x \in M$, $\gamma \in \Gamma$ and $(M\Gamma x\gamma x\Gamma M)$ is semiprime, we have $x \in (M\Gamma x\gamma x\Gamma M)$, so $M$ is intra-regular. $\square$

### 3 On left regular and left duo \textit{po}-\textit{\(\Gamma\)}-semigroups

First we notice that the left (and the right) regular \textit{po}-\textit{\(\Gamma\)}-semigroups are intra-regular. Then we characterize the \textit{po}-\textit{\(\Gamma\)}-semigroups which are both left regular and left duo in terms of filters and we prove that a \textit{po}-\textit{\(\Gamma\)}-semigroup $M$ is left (resp. right) regular if and only if the left (resp. right) ideals of $M$ are semiprime.

**Definition 5.** An ordered $\Gamma$-semigroup $M$ is called left regular (resp. right regular) if

$$x \in (M\Gamma x\gamma x) \text{ (resp. } x \in (x\gamma x\Gamma M))$$
for every \( x \in M \) and every \( \gamma \in \Gamma \).

**Proposition 6.** Let \( M \) be an ordered \( \Gamma \)-semigroup. If \( M \) is left (resp. right) regular, then \( M \) is intra-regular.

**Proof.** Let \( M \) be left regular, \( x \in M \) and \( \gamma \in \Gamma \). Then we have
\[
\begin{align*}
x \in (M \Gamma x \gamma x) & \subseteq \left( M \Gamma (M \Gamma x \gamma x) \gamma x \right) \\
& = \left( M \Gamma (M \Gamma x \gamma x) \gamma x \right) \\
& \subseteq \left( (M \Gamma M) \Gamma (x \gamma x) \Gamma M \right) \subseteq \left( M \Gamma x \gamma x \Gamma M \right),
\end{align*}
\]
thus \( M \) is intra-regular. Similarly, the right regular po-\( \Gamma \)-semigroups are intra-regular. \( \square \)

**Theorem 7.** An ordered \( \Gamma \)-semigroup \( M \) is left regular and left duo if and only if, for every \( x \in M \), we have
\[
N(x) = \{ y \in M \mid x \in (M \Gamma y) \}.
\]

**Proof.** \( \implies \). Let \( x \in M \) and \( T := \{ y \in M \mid x \in (M \Gamma y) \} \). Since \( M \) is left regular, we have \( x \in (M \Gamma x \gamma x) \subseteq \left( (M \Gamma M) \Gamma x \right) \subseteq (M \Gamma x) \), so \( x \in T \), and \( T \) is a nonempty subset of \( M \).

Let \( a, b \in T \) and \( \gamma \in \Gamma \). Since \( x \in (M \Gamma a) \), \( x \in (M \Gamma b) \) and \( M \) is left regular, we have
\[
\begin{align*}
x \in (M \Gamma x \gamma x) & \subseteq \left( M \Gamma (M \Gamma b) \gamma (M \Gamma a) \right) \\
& = \left( M \Gamma (M \Gamma b) \gamma (M \Gamma a) \right) \\
& \subseteq \left( M \Gamma (b \gamma M \Gamma a) \right).
\end{align*}
\]
In addition, \( b \gamma M \Gamma a \subseteq (M \Gamma a \gamma b) \). Indeed: Let \( b \gamma u \mu a \in b \gamma M \Gamma a \), where \( u \in M \) and \( \mu \in \Gamma \). Since \( M \) is left regular, we have
\[
b \gamma u \mu a \in \left( M \Gamma (b \gamma u \mu a) \gamma (b \gamma u \mu a) \right) \subseteq \left( M \Gamma (a \gamma b) \Gamma M \right) = \left( (M \Gamma a \gamma b) \Gamma M \right).
\]
Since \( (M \Gamma a \gamma b) \) is a left ideal of \( M \), it is a right ideal of \( M \) as well, so \( (M \Gamma a \gamma b) \Gamma M \subseteq (M \Gamma a \gamma b) \), then \( b \gamma u \mu a \in \left( (M \Gamma a \gamma b) \right) = (M \Gamma a \gamma b) \). Hence we obtain
\[
x \in \left( M \Gamma (M \Gamma a \gamma b) \right) = \left( M \Gamma (M \Gamma a \gamma b) \right) \subseteq \left( M \Gamma (a \gamma b) \right),
\]
from which \( a \gamma b \in T \).

Let \( a, b \in M \) and \( \gamma \in \Gamma \) such that \( a \gamma b \in T \). Since \( x \in (M \Gamma a \gamma b) \subseteq (M \Gamma b) \), we have \( b \in T \). Besides, \( x \in (M \Gamma a \gamma b) \subseteq \left( (M \Gamma a) \Gamma M \right) \). The set \( (M \Gamma a) \) as a left ideal of \( M \), it is a right ideal of \( M \) as well, so \( (M \Gamma a) \Gamma M \subseteq (M \Gamma a) \). Thus we have \( x \in \left( (M \Gamma a) \right) = (M \Gamma a) \), and \( a \in T \).

Let \( a \in T \) and \( M \ni b \ni a \). Then we have \( x \in (M \Gamma a) \subseteq (M \Gamma b) \), so \( b \in T \).
Let \( F \) be a filter of \( M \) such that \( x \in F \) and let \( a \in T \). Since \( x \in (M\Gamma a) \), we have \( F \ni x \leq u\mu a \) for some \( u \in M, \mu \in \Gamma \). Since \( F \) is a filter of \( M \), we have \( u\mu a \in F \), and \( a \in F \).

\[ \iff \]

Let \( x \in M \) and \( \gamma \in \Gamma \). Since \( x \in N(x) \) and \( N(x) \) is a subsemigroup of \( M \), we have \( x\gamma x \in N(x) \). By hypothesis, we get \( x \in (M\Gamma x\gamma x) \), so \( M \) is left regular. Let now \( A \) be a left ideal of \( M \), \( a \in A \), \( \gamma \in \Gamma \), and \( u \in M \).

Since \( a\gamma u \in N(a\gamma u) \) and \( N(a\gamma u) \) is a filter of \( M \), we have \( a \in N(a\gamma u) \). By hypothesis, we have \( a\gamma u \in (M\Gamma a) \subseteq (M\Gamma A) \subseteq (A) = A \). Thus \( A \) is right ideal of \( M \), and \( M \) is left duo. \( \square \)

The right analogue of Theorem 7 also holds, and we have

**Theorem 8.** An ordered \( \Gamma \)-semigroup \( M \) is right regular and right duo if and only if, for every \( x \in M \), we have

\[ N(x) = \{ y \in M \mid x \in (y\Gamma M) \} \]

**Theorem 9.** An ordered \( \Gamma \)-semigroup \( M \) is left (resp. right) regular if and only if the left (resp. right) ideals of \( M \) are semiprime.

**Proof.** \( \implies \). Let \( M \) be left regular, \( A \) a left ideal of \( M \), \( x \in M \) and \( \gamma \in \Gamma \) such that \( x\gamma x \in A \). Then we have \( x \in (M\Gamma x\gamma x) \subseteq (M\Gamma A) \subseteq (A) = A \), so \( M \) is semiprime.

\( \iff \). Suppose the left ideals of \( M \) are semiprime and let \( x \in M \) and \( \gamma \in \Gamma \). Since \( (M\Gamma x\gamma x) \) is a left ideal of \( M \), \( x\gamma x \in M \), \( \gamma \in \Gamma \) and \( (x\gamma x)\gamma(x\gamma x) \in (M\Gamma x\gamma x) \), we have \( x\gamma x \in (M\Gamma x\gamma x) \). Again since \( (M\Gamma x\gamma x) \) is semiprime, \( x \in M \), \( \gamma \in \Gamma \) and \( x\gamma x \in (M\Gamma x\gamma x) \), we have \( x \in (M\Gamma x\gamma x) \), thus \( M \) is left regular. In a similar way we prove that \( M \) is right regular. \( \square \)

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